On the distributed synchronization of on-line IIM Interdependency Models

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***Abstract*— In the last few years Critical Infrastructures have become more and more tightly interconnected and their protection is one of the major issues for the national and international security. In order to achieve that, many modeling techniques for the interdependencies existing among them have been proposed. Now, a crucial issue is how to use such models to develop tools able to estimate the status of key elements of CIs, quantify the possible threats and suggest adequate countermeasures to human operators and actors. Due to security, commercial and technological aspects, the only feasible approach is to provide distributed and interconnected state/interdependency estimators. In this paper the general problem of the state estimation of interconnected systems sharing the same model is introduced. A first step in the solution of such a challenging problem is then provided in the case of linear systems. The final objective of this research is to define an effective framework for the problem at hand, and then implement and validate an on-line distributed state/interdependency estimator within the EU IST MICIE project.**

I. Introduction

In order to improve the security of national and international Critical Infrastructures (CIs) it is mandatory to define and adopt adequate control actions and countermeasures to assess and reduce vulnerabilities to potential threats and cascading failures. The risk reduction that will be achieved will be mainly related to the capabiliy of a distributed tool to predict failures in one infrastructure due to dependencies from other infrastructures. The same tool, then, should be able to predict the effectivenes of a recovery strategy that usually is adopted in the hypotesis of no failures outside the affected infrastructure.

To this end a very first step is the definition of interdependency models, able to capture the most characterizing aspects of the behavior of interconnected infrastructures (and their elements), either in normal and critical conditions. Online tools able to identify and quantify possible threats and prompt the operators will be based on such models of the System of Systems. The main obstacle, however, is that usually the control centers of the CIs are not interconnected, and the policies and countermeasures adopted by different stakeholders are often limited as a consequence of their narrow point of view. Indeed, the problem is relevant because there is the need to share information and policies among the infrastructure's owners and, at the same time, this shared information must obviously be partial and limited, for security, commercial and technological issues. Moreover, there is the need to design a consistent and reliable state estimator; in fact, as exposed above, an on-line tool must necessarily be a *distributed* tool, and many synchronization issues may arise, due to the delays introduced by the local elaboration and transmission of the information and due to the partiality of the information being exchanged. The design and implementation of a tool with the above characteristics is the main objective of the EU IST project MICIE [14].

In order to solve the major issues of this problem, in this paper the authors provide a general definition of the problem itself. Then, a first step in the solution of such problem is given, referring to the distributed state estimation in the case of *linear* interdependency models with complete information; in fact the *Input-Output Inoperability Model* (IIM) is adopted, in order to provide an initial proof of the effectiveness of the proposed approach.

The paper is organized as follows: in section II the main issues related to the interdependencies among Critical Infrastructures and their composing elements are briefly detailed and the Input-Output Inoperability Method is introduced. In section III the problem of the distributed on-line state estimation is discussed and a general framework for the solution of such challenging problem is exposed. In section IV the general problem is specified in the case of linear IIM distributed estimators with complete information, while in section V a little but significant example is provided. Finally, some conclusive remarks and directions for future work are collected in section VI.

II. Interdependency Modeling

In the last years, a relevant and rapid change in the structure of the national infrastructures (e.g., energy production/distribution, telecommunication networks, transportations etc. [6]) has been observed; in fact, such infrastructures have become more and more tightly interconnected and interdependent, mostly because of the huge diffusion of Information and Communication Technologies (ICT), and also in order to reduce operative costs.

The emerging System of Systems (SoS) seems to be weak against adverse events; in fact, due to the peculiar topological structure that such kind of system may have

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[27], [28], a negative event may easily propagate its effects [29], generating cascading failures that may eventually cross the boundaries of the single infrastructure, provoking huge economical damages and affecting a large segment of the population, as dramatically highlighted by the black-outs that affected USA and Canada in 2003 [3]. Moreover, due to their vulnerabilities, Critical Infrastructures may become targets for terrorist and criminals actions [11]. Indeed any critical infrastructure is a complex, highly non-linear, geographically dispersed cluster of systems; the interactions with other infrastructures and with owners, operators and users further complicate the network of relations that drive its existence.

The protection of CIs is then one of the fundamental challenges, in order to grant national and international security and welfare [17]. The aim of *Critical Infrastructure Protection* (CIP) is then to avoid the above threats (or at least mitigate their impact and effects), developing and implementing adequate strategies and countermeasures [6]; therefore, there is the need to perform analytical, simulative and what-if analyses, in order to identify and understand the global behavior of these systems and highlight weaknesses of infrastructures and their components.

In the literature, many approaches have been introduced in order to address such a challenging complexity [10], [15], [19], [20], [21]; in this paper we will refer to the simple, yet powerful, *Input-Output Inoperability Model* (IIM), because it is linear, and then particularly suitable for the definition of an initial distributed state estimation framework.

A. IIM Interdependency Model

The IIM model main objective is to represent within a simple framework the global effects of negative events in an highly interdependent scenario. Such an approach can be adopted for the representation of the interdependencies among infrastructures considered as monolithic entities, or depict a more detailed scenario: in fact, every infrastructure may be decomposed into a web of interconnected entities with the desired level of abstraction. In this way the boundaries of each infrastructure tend to fade, while it is possible to specify cross-infrastructure interdependencies among elements.

In order to provide an indicator of the state of each infrastructure or element, the *inoperability* is introduced, as the inability (in percentage) to correctly perform its intended functions and operations. This model highlights how the inoperability of each element influences the others, and how such indicator is propagated and eventually amplified, because of the interdependencies. To this end this model assumes that the percentage of inoperability of each infrastructure (or element) depends, by means of given correlation terms, the *Leontief coefficients*, on the level of inoperability of other infrastructures and on failures induced by an external cause.

The Inoperability Input-Output Model can be described in its more general version by the following non-linear system:

$$\dot{x}(t) = f\big(x_1(t), \ldots, x_m(t), c(t)\big) \tag{1}$$

where x_i is the percentage of inoperability of $i - th$ infrastructure and $c(t)$ represents the external causes. The above model can be approximated with the classic linear Leontief dynamic I-O model given in [32]:

$$\dot{x}(t) = \mathcal{A}\,x(t) + c(t) - x(t) \tag{2}$$

The equilibrium point of the system represents the final consequences of an induced negative event on the overall system.

$$x_{eq} = (I - \mathcal{A})^{-1} c(t) \tag{3}$$

This formulation is quite simple, but is useful to depict the cascading effects that may affect interconnected infrastructures and their elements.

One of the most relevant issues is the evaluation of the Leontief coefficients. In [10] and [7], they are derived from economical and statistic data. A different approach has been recently proposed in [18], where these macroscopic coefficients are calculated on the base of the correlation existing among each couple of infrastructures, these latter evaluated using topological-base simulations. Another crucial problem, however, is how to use interdependency models and analysis methodologies as a tool to help operators and stakeholders to become aware of the status of the overall system and its parts and then define adequate countermeasures to cascading negative events.

III. On-line Distributed State Estimation

The on-line state estimation is a crucial aspect and is the final objective of the methodologies that have been developed for the analysis and representation of the interdependencies among critical infrastructures and their elements. In fact, in order to prove the effectiveness of the models that have been proposed in the literature, there is the need to provide on-line tools able to notice the actual effects of negative phenomena, calculate the evolution of the state of the overall system, and then support the decisions of human operators and actors.

The models proposed in the literature are mostly aimed to depict the overall system of system with a global, yet articulated, perspective (for example [15]) , or by means of the federation of the isolated simulators of the different infrastructures [31], [21]. However, such modeling techniques do not take into account the issues of a decentralized perspective. In fact, the implementation of a centralized tool seems not feasible, because, typically, each infrastructure has its own isolated control room, equipped with an estimator of the internal state of the infrastructure itself. Moreover

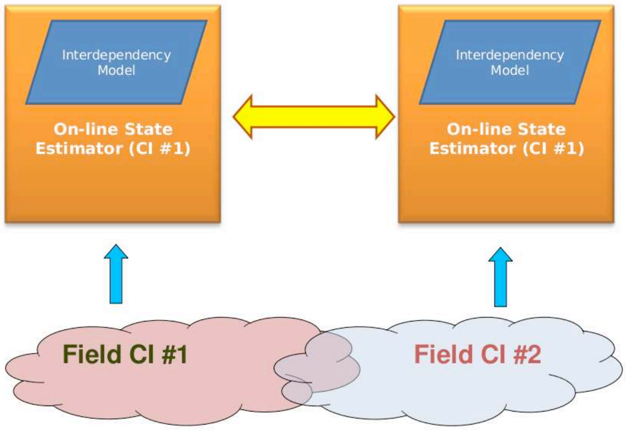

Fig. 1. Example of Distributed Estimators of the state of two interconnected Critical Infrastructures. Each estimator has the same model of the system of systems, but every tool receives directly only the inputs within its own field.

the definition of a centralized state estimator requires that such tool have the complete knowledge of status of every infrastructure and their parts; such requirement is not easy to satisfy, due to the huge amount of data that must be taken into account and because of the obvious security aspects related to the disclosure of critical information.

A more feasible compromise may be a de-centralized, yet synchronized, scenario; from such a perspective, each control center can be equipped with a global model of the overall system of systems (see Figure 1). Obviously the tool inside each infrastructure directly receives only the data originated within such infrastructure. Moreover the different tools must be interconnected. The main issue is then the synchronization of such models. In our view the easiest way to grant the consistency of the overall state estimated by the different distributed tools is to equip every system with a common general model, even if every specific domain receives only a subset of the inputs. Hence we propose that every tool has *exactly the same* model of the overall system of systems (see Figure 1).

In the following section this general framework will be detailed considering the IIM model.

IV. Synchronization of Interconnected Systems

Let us consider the linear IIM model described in (2) where terms are re-arranged as follows for sake of simplicity:

$$\dot{x}(t) = A\,x(t) + B\,u(t) \tag{4}$$

with $A = (\mathcal{A} - I)$ and $B\,u(t) = c(t)$. In the proposed scenario, where several interconnected systems are considered, the original monolithic model is replaced by n identical interconnected models. In particular, when considering interaction the dynamic of the $i-th$ becomes:

$$\dot{x}_i(t) = A\,x_i(t) + B\,u_i + \sum_{j\in\mathcal{N}_i}\big(x_j(t) - x_i(t)\big) \tag{5}$$

where $x_i = \big[x_{i,1}, \ldots, x_{i,m}\big] \in R^m$, $u_i = \big[u_{i,1}, \ldots, u_{i,q}\big] \in R^q$ are respectively the state and the input of the $i-th$ system and $\mathcal{N}_i$ is the subset of systems to which the $i-th$ system s connected to.

The overall dynamic of the set of interconnected systems can be summarized as follows:

$$\dot{x}(t) = \big(I_n \otimes A\big)\,x(t) + \big(I_n \otimes B\big)\,u - \big(L \otimes D\big)\,x(t) \tag{6}$$

where $x = \big[x_1, \ldots, x_n\big]^T$, $u = \big[u_1, \ldots, u_n\big]^T$, $\otimes$ is the Kronecker product [33], $I_n \in R^{n\times n}$ is the identity matrix, L is the Laplacian matrix [34], describing the interconnection among systems and $D \in R^{m\times m}$ describes the coupling between the state vectors of two systems (in the following analysis we assume $D = I_m$).

The problem of reaching synchronization among dynamical systems has been thoroughly investigated over the years. Indeed, several results concerning synchronization in complex systems for both linear and non-linear dynamical systems can be found in literature. Among the others, the reader is referred to [24] for a comprehensive overview about the state of the art of this research topic. In particular, we are interested in reviewing conditions under which the system described in (6) reaches synchronization.

Theorem 4.1: The interconnected system described in (6) is stable if the dynamic matrix A of each system is stable.

Proof: This property can be easily proved by showing that the dynamic matrix of the system $\hat{A} = \big(I \otimes A\big) - (L \otimes D)$ is a negative-definite diagonally dominant matrix if the dynamic matrix A of each system is stable. For sake of simplicity and without any lost of generality, we consider the dynamic system given in (5) to have a 1-dimensional state space, hence the dynamic matrix A is a scalar value a as well as the coupling matrix D, which becomes a scalar value 1. In this case, the overall system can be simply written as follows:

$$\dot{x}(t) = \tilde{A}\,x(t) + \tilde{B}\,u(t) - \tilde{L}\,x(t)$$

where $\tilde{A} = a \cdot I_n$, $\tilde{B} = I_n \otimes B$ and $\tilde{L} = L$. At this point, in order to prove that the dynamic matrix $\hat{A} = \tilde{A} - \tilde{L}$ is diagonally dominant, the following conditions must be fulfilled for each row:

$$\mid \hat{a}_{ii} \mid > \sum_{j\neq i} \mid \hat{a}_{ij} \mid .$$

Looking at the structure of the matrix $\hat{A}$, we have the following inequality for the $i-th$ row:

$$| a - d_i | > | d_i | \quad \forall i \in (1, \ldots n)$$

where d_i is the cardinality of the neighborhood $\mathcal{N}_i$ of the $i-th$ system and hence $d_i \geq 0$ by definition. As a result, in order to have a diagonally dominant matrix, it must be:

$$a < 0 \quad \vee \quad a > \max_{i \in (1, \ldots n)} \{2\, d_i\}.$$

At this point, by exploiting the Gershgorin circle theorem [22], it follows that the overall dynamic of the system is stable if $a < 0$, thus proving the statement.

Note that, this proof can be properly extended to the case of a dynamical system A with a n-dimensional state-space by exploiting the generalization of the Gershgorin circle theorem to block diagonally dominant matrices given in [23]. ■

Theorem 4.2: A sufficient condition for the system described in (6) to reach synchronization is that the set of inputs $u = \left[u_1, \ldots, u_n\right]^T$ converges to the same value.

Proof: This property can be easily proved by exploiting the result of theorem 4.1 and observing that such a definite-negative dominant dynamic matrix is non-singular and by construction all rows sum to the same value.

As in theorem 4.1, for sake of simplicity and without any lost of generality, a dynamic system with 1-dimensional state-space is considered in (5), with $A = a$ and $D = 1$. Therefore, the overall system can be written again as:

$$\dot{x}(t) = \tilde{A}\, x(t) + \tilde{B}\, u(t) - \tilde{L}\, x(t).$$

At this point, as the dynamic matrix $\hat{A} = \tilde{A} - \tilde{L}$ under the condition given in theorem 4.1 is non-singular, for any given set of inputs u an equilibrium is always reached. Therefore, let us consider the situation at such an equilibrium:

$$0 = \hat{A}\, x + \tilde{B}\, u$$

which can be re-arranged as follows:

$$\hat{A}\, x = -\tilde{B}\, u.$$

Now, as by construction all rows of $\hat{A}$ sum to $a \cdot \mathbf{1}_n^T$, synchronization is reached for any set of inputs $u = \left[u_1, \ldots, u_n\right]^T$ such that:

$$\tilde{B}\, u \in \text{span}\{\mathbf{1}_n^T\}.$$

At this point, it is quite straightforward to notice that the above condition can be easily fulfilled by assuming:

$$u_1 = u_2 = \ldots = u_n = \bar{u},$$

proving the statement. ■

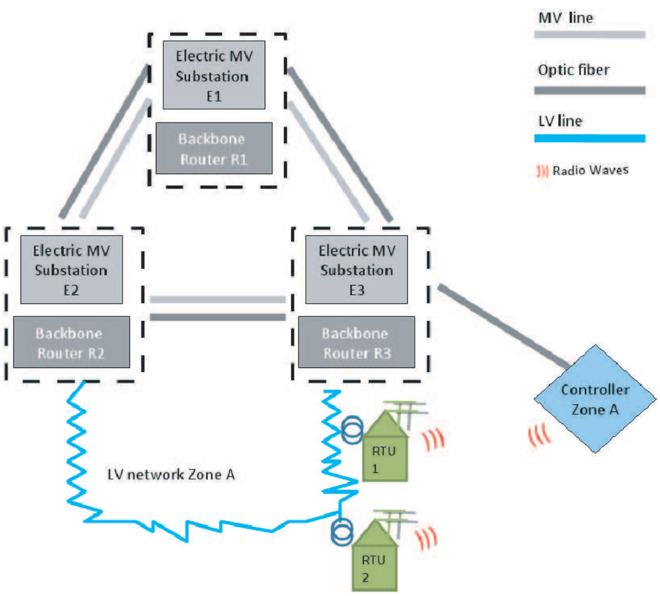

Fig. 2. A simple Case Study composed of a portion of the Electrical, Telecommunication and Scada Infrastructures.

So far, conditions under which the system described in (6) reaches synchronization were investigated. At this point, a technique by which the inputs of the interconnected systems can converge to the same value is required. A suitable modeling to solve this problem in a distributed way is provided by the consensus filtering. In particular, the max-consensus algorithm given in [30] provides a simple and effective way to reach agreement to the max value of a variable of interest over a network. By exploiting this result, a theorem for convergence to the same value for the set of inputs $u = \left[u_1, \ldots, u_n\right]^T$ associated to the system described in (6) is given.

Theorem 4.3: The max-consensus algorithm for the set of inputs $u = \left[u_1, \ldots, u_n\right]^T$ associated to the system described in (6) converges to a common value $\bar{u}$, that is:

$$u_1 = u_2 = \ldots = u_n = \bar{u},$$

where

$$\bar{u} = [\bar{u}_1, \ldots, \bar{u}_q], \qquad \text{with} \quad \bar{u}_i = \max_{j \in (1, \ldots, n)} \{u_{j,i}\}$$

if the network topology is connected.

Proof: The proof of the scalar version of this problem formulation for a digraph is given in [30] (Section 8). The simpler version involving a network topology described by an undirected graph requires the graph simply to be connected. Moreover, the extension to a vectorial version is trivial. ■

Theorem 4.3 has the following interpretation for the proposed scenario. Let us recall that the input $u_i \in R^q$ for the $i-th$ system is a vector of binary variables describing

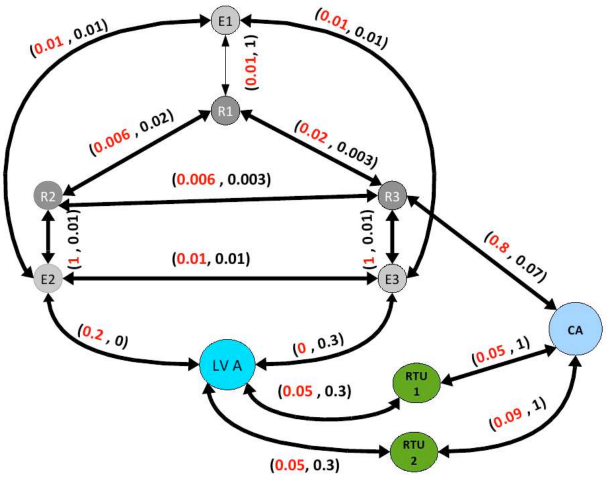

Fig. 3. Example of IIM model. The Leontief coefficients are synthetically represented over the arcs

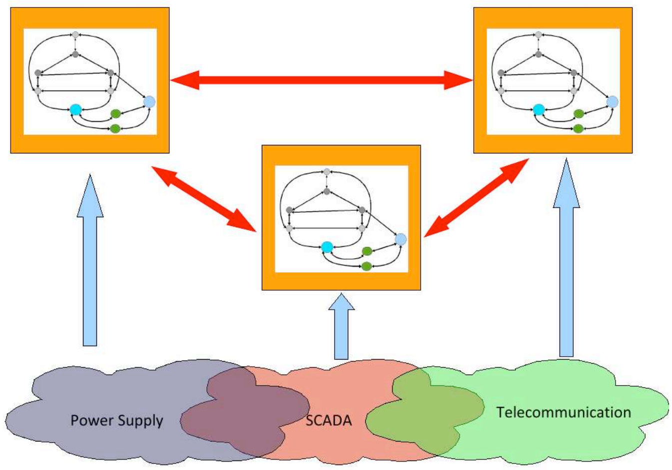

Fig. 4. Distributed and interconnected state estimators for the system defined in the case study.

whether a certain negative event j happens ($u_{i,j} = 1$) or not ($u_{i,j} = 0$). At this point, by means of the max-consensus, each system is locally able to have a global picture of the situation, and then all the estimators can evolve synchronously (according to theorem 4.2) as if all the negative events were available to a central unit.

V. Case Study

In Figure 2 a small segment of an high voltage (HV) and medium voltage (MV) electric grid is depicted; a part of the telecommunication and Scada infrastructures used for the remote control are also represented.

More in detail, the HV grid is composed by several substations (S1, S2 and S3), connected with a ring topology. In order to provide adequate control to the infrastructure, there is an high speed telecommunication backbone, whose routers (R1 R2 and R3) are in spatial proximity with the corresponding substation. The Portion of the Scada system is composed of two RTUs and a control center.

In Figure 3 the resulting IIM model for the above scenario is represented; the Leontief coefficients are synthetically represented over the arcs; if two elements x and y are interrelated the corresponding Leontief coefficients are (a_{xy}, a_{yx}).

The resulting framework is represented in Figure 4; the general IIM model is reproduced within the electrical, Scada and telecommunication infrastructures.

VI. Conclusions and Future Work

In this paper the general problem of the synchronization of multiple distributed tools with a common model of the interdependencies existing among CI's and their elements has been introduced. In particular the problem has been specified in the case of linear IIM models.

Further researches will be devoted to develop a framework for the synchronization of distributed state estimators that rely on a common model composed of different, yet interrelated, levels of abstraction [25], [26]. This is one of the main objectives of the EU IST project MICIE[14]; the project will provide an on-line risk predictor/state estimator composed of distributed tools, connected by means of a secure and reliable transmission line. The tool will be validated on both a telecommunication and electrical infrastructures.

In this paper we have addressed the problem in the case of complete information; however, as stressed above, the data exchange among different infrastructures is necessarily partial; therefore an open issue is how to ensure the synchronization of multiple state estimators by means of the exchange of partial information.

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