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Agent-based input–output interdependency model

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ABSTRACT

The modeling and analysis of critical infrastructures and their interdependencies are essential to discovering hidden vulnerabilities and the related threats to national and international security. Over the past few years, several approaches have been proposed to address this problem. The so-called holistic approaches are relatively abstract, but are easily validated using real economic data. Other approaches based on agent-based models provide deeper views of the interdependencies existing between subsystems of different infrastructures. However, agent-based models are often difficult to validate because quantitative data of the appropriate granularity may not be available.

This paper presents an agent-based input–output inoperability model designed to overcome the limitations of the holistic and agent-based paradigms. In order to provide a detailed and expressive framework, the exchange of resources between infrastructures is explicitly modeled while inoperability becomes an internal parameter. Nevertheless, the model is easily transformed into a fine-grained, input–output inoperability model whose coefficients can be obtained based on real data.

1. Introduction

Infrastructures such as energy grids, transportation networks and telecommunications systems are critical to the welfare, economy and security of every developed country. Protecting these critical infrastructures is one of the main challenges for governments and international organizations [1].

In order to develop robust infrastructure protection strategies, it is important to identify and understand the global behavior and intrinsic weaknesses of these systems and their components, especially in the face of adverse events. Several systems analysis and simulation approaches have been proposed for this purpose. However, they are limited in their effectiveness [2], mainly because of the overwhelming complexity of critical infrastructures [3]. In particular, it is necessary to model and analyze the mutual relationships that exist within an infrastructure and between infrastructures [4]. These dependencies are often implicit, hidden and poorly understood by infrastructure owners and operators as well as domain experts. Moreover, as the level of detail of the critical infrastructure modeling framework is increased, the model becomes harder to validate primarily due to the lack of quantitative data of the appropriate granularity.

This paper describes a detailed input–output model that addresses the limitations of other approaches. It decomposes infrastructures into subsystems and considers the exchange of different resources (goods or services) between subsystems. The model is easily transformed into the input–output inoperability model (IIM) [5], where the model coefficients are based on the knowledge and experience of infrastructure stakeholders and domain experts.

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2. Interdependency modeling

The modeling of interdependencies existing among infrastructures and subsystems is crucial to identifying the structural vulnerabilities and quantifying threats. Several modeling approaches have been proposed in the literature. The most popular approach involves the use of holistic models where each infrastructure is represented as a unique, monolithic entity.

Another approach that has gained attention is the Input–Output Interdependency Model (IIM) [5], which captures interdependencies based on the economic interactions between infrastructures. The IIM has several extensions such as the Dynamic Input–Output Model (D-IIM) [6], which considers system dynamics during crisis situations; and the Multi-Regional IIM (MR-IIM) [7], which can represent multi-sector and multi-regional economic interdependencies.

In the traditional IIM, interactions between infrastructures are modeled at a high level of abstraction (Fig. 1), which tends to mask the low level dependencies. Moreover, the high level of abstraction introduces a simplification that ignores the structural and geographical aspects of infrastructures. In fact, a critical infrastructure is a complex, geographically-dispersed cluster of systems whose mutual interactions greatly influence the overall behavior. As a consequence, it is necessary to decompose each infrastructure into a web of interconnected entities whose links may cross the boundaries of individual infrastructures.

This new decomposition approach, called Agent-Based Modeling and Simulation (ABMS) [8], is illustrated in Fig. 2. An ABMS framework uses elementary, interconnected software agents to represent infrastructures and their components, providing insight into the behavior of each system and its parts and enabling interdependencies to be modeled with fine granularity. The approach does not impose any limits on the granularity used to describe or decompose infrastructures, providing a flexible and versatile modeling framework (see, e.g., [9–12]).

The main strength of the IIM approach is the availability of economic data that can be used to validate the model. On the other hand, the finer granularity of ABMS models makes them difficult to validate. Very little quantitative data of the appropriate granularity is available; the only choice is to obtain subjective data from asset owners, operators and domain experts.

In recent years, critical infrastructures have attained high degrees of interoperability, mainly due to pervasive Internet technologies. However, because of the increased complexity of infrastructures, technicians and operators have become more specialized and sector-specific. Therefore, while it is possible to obtain detailed information about individual infrastructures and infrastructure elements, cross-infrastructure interdependencies are often implicit, hidden and/or not well understood by experts.

At first glance, it appears to be feasible to adopt the IIM approach at a low level of abstraction, decompose each infrastructure into a set of interconnected elements and consider their inoperabilities and their mutual influence. However, such a representation is abstract and incomplete because the operativeness of infrastructures and their elements rely on the production, exchange and consumption of resources. On the other hand, the quantification of interdependencies in an agent-based approach can be very arbitrary. Therefore, our strategy is to combine the two approaches in an agent-based IIM.

3. Input–Output inoperability model

The main objective of the Input–Output Inoperability Model (IIM) [5] and its refinement [6] is to represent within a simple framework the global effects of negative events in scenarios involving highly interdependent infrastructures. The model, which is based on economic data, analyses how the effects of natural outages or terrorist attacks on one infrastructure may affect other infrastructures while also highlighting the cascading effects and intrinsic vulnerabilities. The principal assumption is that two entities that have substantial economic interaction will have a large amount of physical interdependence [6].

Infrastructure interactions can be estimated in a variety of ways. However, in the case of the United States, the most reliable data sources are the Bureau of Economic Analysis (BEA) National I-O Accounts Database and the Regional Input–Output Multiplier System. The BEA database, in particular, provides tables depicting the production and consumption of commodities (goods and resources) by various sectors of the US economy. This data is used to compute the Leontief technical coefficients used in the IIM framework.

In the original Leontief model, each industry is assumed to produce a single commodity. Since this assumption is not realistic, the BEA considers different commodities for each industry and provides two data matrices: the “industry by commodity” matrix and the “commodity by industry” matrix [13]. These matrices, which are often referred to as the “make” and “use” matrices, are used to derive the input–output matrix [6,14].

Another relevant assumption in the IIM is that the technical coefficients are constant. This implies that the prices and means of production remain fixed [15].

In order to deal with interdependencies between critical infrastructures, the original economic model is transformed by normalizing the sector outputs and demands to variables defined on scales that vary depending on the sector outputs. Note that the terms industry, economic sector and critical infrastructure are not synonyms. However, due to the crudeness of the economic approximation, the technical coefficients in IIM are derived from economic data by considering a scenario to involve n critical infrastructures, where each infrastructure is correlated to a given economic sector.

Finally, the existence of an equilibrium is an important assumption for the dynamic extension of the input–output economic model and the IIM.

This section examines the static and dynamic versions of IIM. A systems engineering perspective is adopted. Thus, the equilibrium of the dynamic IIM is not assumed *a priori*; instead, the dynamical system is examined, the conditions for the existence of the equilibrium are provided along with its correspondence with the equilibrium reached in the static IIM.

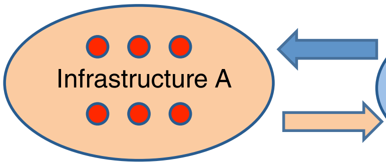

Fig. 1 – Monolithic IIM representation of interdependent infrastructures.

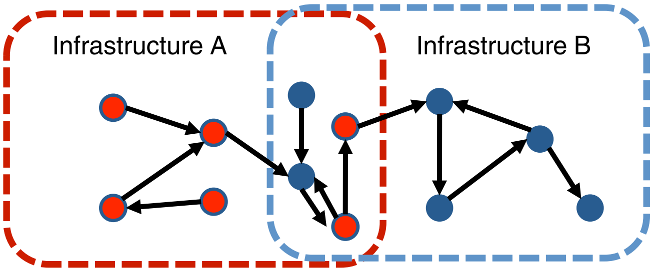

Fig. 2 – ABMS decomposition of interdependent infrastructures.

3.1. *Notation*

In this paper, a vector is typically expressed using a lowercase letter in boldface. Lowercase letters are used to denote scalar values. A matrix Z is represented by an uppercase italicized letter and its elements are denoted as z_{ij}.

I_n denotes the $n \times n$ identity matrix and $\mathbf{1_p}$ is a vector with p components, each of which is equal to one. The ith row of matrix A is denoted by $row_i\{A\}$

A linear dynamical system has the form:

$$\dot{\mathbf{g}}(t) = Z\mathbf{g}(t) + H\mathbf{u}(t) \tag{1}$$

where $\mathbf{g} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$, Z is an $n \times n$ matrix and H is an $n \times m$ matrix.

The evolution of the dynamical system is given by:

$$\mathbf{g}(t) = e^{Zt}\mathbf{g}(0) + \int_0^t e^{Z(t-\tau)} Hu(\tau)d\tau = \mathbf{g_f}(t) + \mathbf{g_u}(t) \tag{2}$$

where $\mathbf{g_f}$ and $\mathbf{g_u}$ are the “free” and “forced” evolutions of the system, respectively [16].

3.2. *Static IIM*

According to the economic equilibrium theory of Leontief [17], a static demand-reduction model [6,5] for n infrastructures is given by:

$$\delta\mathbf{x} = A\delta\mathbf{x} + \delta\mathbf{c} \tag{3}$$

where $\delta\mathbf{x}$ is defined as the difference between the as-planned ($\mathbf{x_0}$) and degraded ($\mathbf{x_d}$) production; $\delta\mathbf{c}$ is the difference between the as-planned ($\mathbf{c_0}$) and degraded ($\mathbf{c_d}$) final demands; and A is an $n \times n$ matrix of Leontief technical coefficients where each element a_{ij} represents the ratio of the input from infrastructure i to infrastructure j with respect to the overall production requirements of infrastructure j.

The “inoperability” of an infrastructure is defined as its inability (expressed as a percentage) to operate correctly. Haimes, et al. [6] introduced the $n \times n$ transform matrix P given by:

$$P = [diag\{\mathbf{x_0}\}]^{-1}. \tag{4}$$

The inoperability is then obtained applying the transformation:

$$\mathbf{q} = P\delta\mathbf{x}. \tag{5}$$

Thus, the input–output inoperability model has the form:

$$\mathbf{q} = A^*\mathbf{q} + \mathbf{c}^* \tag{6}$$

where $A^* = PAP^{-1}$ and $\mathbf{c}^* = P\mathbf{c}$. Note that $\mathbf{c}^*$ assumes the role of an externally-induced inoperability and can be viewed as a perturbation created by an adverse or malicious event. For a perturbation $\mathbf{c}^*$, the inoperability $\mathbf{q}$ is given by:

$$\mathbf{q} = (I - A^*)^{-1}\mathbf{c}^*. \tag{7}$$

While it is mathematically possible to have $a_{ij} > 1$, the a^*_{ij} coefficients are strictly less than one; moreover, the sum of each column is less than one [14]. Setola, et al. [18] introduced a dependency index γ_i, which is the sum of the IIM coefficients along a row:

$$\gamma_i = \sum_{j=1}^{n} a^*_{ij}. \tag{8}$$

The dependency index expresses the robustness of an infrastructure with respect to the inoperability of the other infrastructures. In fact, it represents the maximum inoperability in infrastructure i when every other infrastructure is fully inoperable. The greater the decrease in γ_i (when $\gamma_i < 1$), the better infrastructure i can maintain its working capabilities (e.g., due to the presence of buffers, back-up power generators, etc.) despite the inoperability of its supplier infrastructures. The dependency index supports a quick global evaluation of the resilience of an infrastructure. Moreover, as will be explained later, it can be used to evaluate the stability of the overall system and, thus, the existence of an equilibrium.

The static IIM can be rearranged to consider “operativeness”, which is defined as:

$$\mathbf{op} = \mathbf{1_n} - \mathbf{q}. \tag{9}$$

The operativeness form of the IIM is given by:

$$\mathbf{op} = A^*\mathbf{op} + \tilde{\mathbf{c}}^* \tag{10}$$

where $\tilde{\mathbf{c}}^* = (I - A^*)\mathbf{1_n} - \mathbf{c}^*$. Note that $\tilde{c_i}^*$ can be expressed as:

$$\tilde{c_i}^* = (1 - c_i^*) - row_i\{A\}\mathbf{1_n} = \hat{c_i}^* - \gamma_i \tag{11}$$

where $\hat{c_i}^*$ is the external induced operativeness and γ_i is the maximum operativeness received by infrastructure i when every other infrastructure is fully operational.

The operativeness of an infrastructure thus depends on the operativeness of the other infrastructures using the same coefficients as the original IIM while considering an induced operativeness and a negative constant balancing term. As will be discussed in Section 4, this form is very convenient for analyzing the relationship between the state of an infrastructure and the production of resources.

3.3. *Dynamic IIM*

This section examines the dynamic extension of the IIM and its relationship to the static model. In addition, conditions for the existence of an equilibrium are derived.

The static IIM defined in Eq. (3) has been extended by incorporating a dynamic term [19]:

$$\delta\mathbf{x}(t) = A\delta\mathbf{x}(t) + \delta\mathbf{c}(t) + B\delta\dot{\mathbf{x}}(t) \tag{12}$$

where B is an $n \times n$ matrix that represents the willingness of the economy to invest in capital resources. Many choices are possible for the B matrix; however, as explained in [19], the elements of B must be either zero or negative for an economic system to be stable.

Haimes, et al. [6] have adopted a diagonal B matrix of the form:

$$B = -K^{-1}; \qquad k_{ii} \geq 0; \quad \forall i = 1, \ldots, n. \tag{13}$$

Upon substituting Eq. (13) in Eq. (12), we obtain:

$$\delta\dot{\mathbf{x}}(t) = K[(A - I)\delta\mathbf{x}(t) + \delta\mathbf{c}(t)]. \tag{14}$$

As in the case of the static demand-reduction model, P can be applied to Eq. (14), leading to the dynamic IIM formulation:

$$\dot{\mathbf{q}}(t) = K(A^* - I)\mathbf{q}(t) + K\mathbf{c}^*(t). \tag{15}$$

Matrix K is the industry resilience coefficient matrix, where each element k_{ii} measures the resilience of infrastructure i. The element k_{ii} can also be viewed as the recovery rate with respect to adverse or malicious events.

The matrix K assumes the role of a control parameter. Implementing countermeasures and risk mitigation strategies for infrastructure i increases k_{ii}, minimizing the economic losses and impact, and leading to shorter recovery times.

The dynamic IIM can be used to represent the response of interdependent infrastructures to an induced perturbation until the equilibrium (if one exists) is reached. The following results provide conditions for system stability and correlate the static and dynamic IIMs.

Lemma 1. *If all the eigenvalues of matrix $(A^* - I)$ have strictly negative real parts and $\|\mathbf{c}^*(t)\|$ is bounded, then the system defined by Eq. (15) is stable. Furthermore, if $\mathbf{c}^*(t)$ is stationary, then the system reaches an equilibrium that coincides with Eq. (7).*

Proof. The evolution of the system is given by $\mathbf{q}(t) = \mathbf{q_f}(t) + \mathbf{q_u}(t)$ where the free evolution $\mathbf{q_f}$ and the forced evolution $\mathbf{q_u}$ are given by:

$$\begin{aligned} \mathbf{q_f}(t) &= e^{K(A^*-I)t}\mathbf{q}(0) \\ \mathbf{q_u}(t) &= \int_0^t e^{K(A^*-I)(t-\tau)}K\mathbf{c}^*(\tau)d\tau. \end{aligned} \tag{16}$$

Since K is a diagonal positive-definite matrix and $(A^* - I)$ is stable, the free evolution of the system converges to zero. Therefore, it is sufficient to prove that $\|\mathbf{q_u}(t)\|$ is bounded. The following inequalities hold:

$$\begin{aligned} \|\mathbf{q_u}(t)\| &\leq \int_0^t \|e^{K(A^*-I)(t-\tau)}\|\|K\mathbf{c}^*(\tau)\|d\tau \\ &\leq \int_0^t e^{-M(t-\tau)}\|K\mathbf{c}^*(\tau)\|d\tau \end{aligned} \tag{17}$$

where M is a positive-definite matrix. Therefore, if $\|K\mathbf{c}^*(\tau)\|$ is bounded, the system defined by Eq. (15) is stable. Since K is a diagonal positive-definite matrix, the following inequality holds:

$$\|K\mathbf{c}^*(\tau)\| \leq k_{max}\|\mathbf{c}^*(t)\|; \qquad k_{max} = \max_{i=1,\ldots,n}\{k_{ii}\}. \tag{18}$$

Since $\|\mathbf{c}^*(t)\|$ is bounded, the system defined by Eq. (15) is stable. Furthermore, if $\mathbf{c}^*(t)$ is stationary, then an equilibrium is reached. This equilibrium is given by:

$$0 = K(A^* - I)\mathbf{q_{eq}}(t) + K\mathbf{c}^* \Rightarrow \mathbf{q_{eq}}(t) = (A^* - I)^{-1}\mathbf{c}^*. \tag{19}$$

Thus, the lemma is proved. □

Lemma 1 provides the first condition for system stability, which is not dependent on the particular matrix K considered. In addition, the lemma defines the relationship between the static and dynamic models.

The following theorem provides an additional condition for system stability.

Theorem 1. *If $\|\mathbf{c}^*(t)\|$ is bounded, then a sufficient condition to guarantee the stability of the system defined by Eq. (15) is that the maximum of the dependency indices γ_i of matrix A^* is less than one.*

Proof. From Lemma 1, if the free evolution is stable, then a bounded $\|\mathbf{c}^*(t)\|$ means that the overall system is stable. Using Gershgorin’s Circle Theorem [20], it follows that the eigenvalues of $K(A^* - I)$ lie (in the complex plane) within the union of circles centered at $k_{ii}(a^*_{ii} - 1)$ with radius equal to $k_{ii}\sum_{j=0;j\neq i}^{n}|a^*_{ij}|$. A necessary condition for stability is given by the inequality:

$$k_{ii}(a^*_{ii} - 1) + k_{ii}\sum_{j=0;j\neq i}^{n}|a^*_{ij}| < 0; \quad \forall i = 1, \ldots, n. \tag{20}$$

Since each $a^*_{ij} \in [0, 1)$, the radius can be expressed as $k_{ii}(\gamma_i - a^*_{ii})$. Therefore, the inequality is verified if $\gamma_i < 1$ for all $i = 1, \ldots, n$. □

Theorem 1 expresses the condition that, if the degree of dependency is sufficiently small, then the increment in inoperability is bounded. Moreover, the presence of buffers, back-up power generators, etc. in each infrastructure ensures that the stability of the system does not depend on K.

3.4. *Discussion*

Although the IIM framework is compact, elegant and capable of modeling cascading effects, the high level of abstraction does not support accurate analyses of the real nature of dependencies. In fact, the framework is suited to analyzing entire infrastructures, not individual subsystems. However, understanding and representing the contributions of individual subsystems are fundamental to dealing with complex, highly-coupled, geographically-dispersed systems. It is, therefore, necessary to decompose infrastructures into a set of more concrete systems. The working capability of these systems is mainly characterized by the availability of resources (goods or services).

The economic origin of IIM also produces a structural limitation. Even when the use and make matrices are considered to account for the production and consumption of multiple commodities by each infrastructure, only data pertaining to the economic value of these commodities is typically available. This data is related to the monolithic perspective of infrastructures.

On the other hand, when decomposing infrastructures based on a finer grain perspective, it is difficult to obtain exact economic data for the resources exchanged between subsystems. The next section describes a methodology that represents interdependencies at a lower level of abstraction and considers the exchange of resources between subsystems.

4. Agent-based IIM

A clear need exists for a more expressive, yet well-grounded, interdependency modeling framework. However, the availability of data is an important issue that must be addressed in order to adopt a finer grain perspective.

The abstract Leontief coefficients can be estimated from macroeconomic data. However, when dealing with more descriptive models, it is not easy to obtain adequate economic data. In fact, when dealing with subsystems, the assumption that interdependency is proportional to economic dependency is not realistic.

Because the IIM is considered to be a standard for dealing with critical infrastructures, it is useful to apply the IIM framework in a reductionistic fashion. Moreover, because economic data is not available while adopting the finer grain perspective, the IIM has to be extended to consider the production, consumption and transmission of resources. This would enable the various stakeholders to contribute to modeling activities and to encode their knowledge in a simple and flexible manner. Such an approach has been engaged by Setola, et al. [18], where the IIM coefficients are estimated using questionnaires and technical interviews.

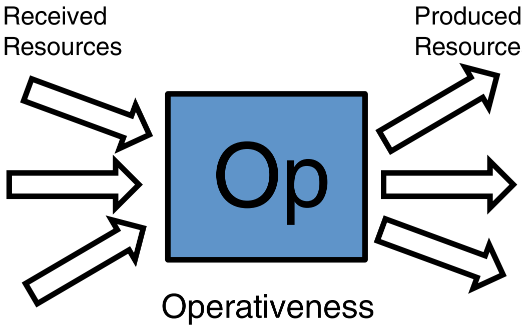

Fig. 3 – Relationships between received resources, operativeness and produced resources.

The Agent-Based IIM (AB-IIM) presented in this paper adopts a different approach. One of the key characteristics is that the (fine-grained) Leontief technical coefficients can be derived quite easily. This feature is very important because the more sophisticated AB-IIM can be used to derive the simpler IIM based on data obtained from stakeholders and domain experts.

4.1. *Static agent-based IIM*

The main idea underlying the AB-IIM is that each element interacts with other elements via the production, exchange and consumption of resources without directly exchanging inoperability. This abstract quantity becomes an internal variable that represents the overall status of each element and drives its dynamics.

An intuitive choice for the model is that the produced resources are proportional to the operativeness, which, in turn, depends on the received resources (Fig. 3). However, in order to reduce the complexity, only the produced resources are directly taken into account. The received resources are taken as a weighted sum of the resources produced by other elements and considering attenuation (i.e., dissipation) during transportation.

Assume that m different resource types exist. Let r_i^j be the normalized production of resource j from element i. This quantity is proportional to the operativeness by means of the coefficient $\phi_i^j \in [0, 1]$:

$$r_i^j = \phi_i^j op_i \Rightarrow \mathbf{r} = \Phi \mathbf{op} \tag{21}$$

where Φ is a $nm \times n$ matrix. On the other hand, the operativeness of an element depends on the weighted sum of the total amount of received resources:

$$op_i = \sum_{j=1}^{m} \psi_i^j \bar{r}_i^j + \tilde{c}_i^* \Rightarrow \mathbf{op} = \Psi \bar{\mathbf{r}} + \tilde{\mathbf{c}}^* \tag{22}$$

where ψ_i^j denotes the influence of resource j received by element i; $\bar{r}_i^j$ is the total amount of resource j received by element i; and Ψ is an $n \times nm$ matrix. The received resources for element i depend on the production of all elements other than i with attenuation δ_{pi}^j during the transmission of resource j from element p to element i:

$$\bar{r}_i^j = \sum_{p=1;p\neq i}^{n} \delta_{pi}^j r_p^j \Rightarrow \bar{\mathbf{r}} = \Delta \mathbf{r} \tag{23}$$

where the Δ is the $nm \times nm$ attenuation matrix. Replacing this equation in Eq. (22) and shifting to the inoperability form yields the static AB-IIM formulation:

$$\begin{bmatrix}\mathbf{q}\\ \mathbf{r}\end{bmatrix} = \begin{bmatrix}0 & -\Psi\Delta\\ -\Phi & 0\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \mathbf{r}\end{bmatrix} + \begin{bmatrix}A^*\\ \Phi\end{bmatrix}\mathbf{1_n} + \begin{bmatrix}I_n\\ 0\end{bmatrix}\mathbf{c}^*. \tag{24}$$

Lemma 2. *If $\Psi\Delta\Phi = A^*$, then the static AB-IIM defined by Eq. (24) coincides with the static IIM defined by Eq. (6).*

Proof. Substituting the second equation of the system defined by Eq. (24) in the first equation yields:

$$\mathbf{q} = \Psi\Delta\Phi\mathbf{q} + (A^* - \Psi\Delta\Phi)\mathbf{1_n} + \mathbf{c}^*. \tag{25}$$

Clearly, if $\Psi\Delta\Phi = A^*$, then the static IIM is obtained. □

In other words, the abstract matrix A^* in the classical IIM is decomposed into the product of three matrices, Φ, Δ and Ψ, each with a physical meaning. Ψ expresses how the availability of different resources influences the operativeness of a single component; Φ expresses the capability of the element to produce its outputs; and Δ expresses the eventual losses due to the links or other factors.

As in the case of the standard IIM, we define certain indices to express the levels of dependency of elements. Let $\Theta = \Psi\Delta$ be an $n \times nm$ matrix whose elements are defined by:

$$\theta_a^b = \sum_{k=1}^{nm} \psi_{ak}\delta_{kb}; \quad a \in [1, n];\ b \in [1, nm]. \tag{26}$$

The AB-IIM dependency index $\bar{\gamma}_i$ for element i is defined as:

$$\bar{\gamma}_i = -\sum_{j=1}^{nm} \theta_i^j. \tag{27}$$

This index represents the cumulative effect of the resources received by element i on its operativeness. The following theorem correlates the AB-IIM dependency index with the dependency index in the standard IIM.

Theorem 2. *If $\Psi\Delta\Phi = A^*$, then $\bar{\gamma}_i$ defined by Eq. (27) coincides with γ_i defined by Eq. (8) for all $i = 1, \ldots, n$.*

Proof. Consider the model described by Eq. (24) for the case of a specific q_i:

$$q_i = row_i\{-\Psi\Delta\}\mathbf{r} + row_i\{A^*\}\mathbf{1_n} + c_i^*. \tag{28}$$

In order to evaluate only the dependency of element i on the other elements, let $c_i^* = 0$. If $\mathbf{r} = 0$, which corresponds to the worst possible scenario, then $q_i = \gamma_i$. Hence, γ_i is the induced inoperability when no resource is received by element i. Conversely, if $r_p^j = \phi_p^j$ for all $p \neq i$, which corresponds to normal working conditions, then $q_i = \gamma_i - \bar{\gamma}_i$. Since the elements are fully operable under normal working conditions, the theorem is proved. □

4.2. *Dynamic agent-based IIM*

Similar to the formulation of the dynamic IIM specified by Eq. (15), we define the dynamic IIM with resource exchange as:

$$\begin{bmatrix}\mathbf{q}(t)\\ \mathbf{r}(t)\end{bmatrix} = \begin{bmatrix}0 & -\Psi\Delta\\ -\Phi & 0\end{bmatrix}\begin{bmatrix}\mathbf{q}(t)\\ \mathbf{r}(t)\end{bmatrix} + \begin{bmatrix}A\\ \Phi\end{bmatrix}\mathbf{1_n} + \begin{bmatrix}I_n\\ 0\end{bmatrix}\mathbf{c}^* + \begin{bmatrix}B\\ D\end{bmatrix}\begin{bmatrix}\dot{\mathbf{q}}(t)\\ \dot{\mathbf{r}}(t)\end{bmatrix} \tag{29}$$

where B is an $n \times n$ matrix defined by Eq. (13) and D is an $nm \times nm$ diagonal matrix defined by:

$$D = -W^{-1}; \qquad w_{ii} \geq 0; \quad \forall i = 1, \ldots, nm. \tag{30}$$

The above model can be rearranged as:

$$\begin{bmatrix}\dot{\mathbf{q}}(t)\\ \dot{\mathbf{r}}(t)\end{bmatrix} = \begin{bmatrix}-K & -K\Psi\Delta\\ -W\Phi & -W\end{bmatrix}\begin{bmatrix}\mathbf{q}(t)\\ \mathbf{r}(t)\end{bmatrix} + \begin{bmatrix}KA\\ W\Phi\end{bmatrix}\mathbf{1_n} + \begin{bmatrix}K\\ 0\end{bmatrix}\mathbf{c}^*(t). \tag{31}$$

Lemma 3. *If the dynamic matrix of the system defined by Eq. (31) has eigenvalues with negative real parts and $\|\mathbf{c}^*(t)\|$ is bounded, then the system is stable. Furthermore, if $A^* = \Psi\Delta\Phi$ and $\mathbf{c}^*(t)$ is stationary, then the system reaches an equilibrium defined by Eq. (7).*

Proof. Since the dynamic matrix is stable, the free evolution converges to zero. As in the case of Lemma 1, it is sufficient to show that

$$\left\| \begin{bmatrix}KA^*\\ W\Phi\end{bmatrix}\mathbf{1_n} + \begin{bmatrix}K\\ 0\end{bmatrix}\mathbf{c}^*(t) \right\| \leq \left\| \begin{bmatrix}KA^*\\ W\Phi\end{bmatrix}\mathbf{1_n} \right\| + \|K\mathbf{c}^*(t)\| \tag{32}$$

is bounded. As in Lemma 1, the result is proved if $\|\mathbf{c}^*(t)\|$ is bounded. Moreover, if $\mathbf{c}^*(t)$ is stationary, at the equilibrium $\dot{\mathbf{q}}(t) = 0$ and $\dot{\mathbf{r}}(t) = 0$, and such an equilibrium coincides with Eq. (24). Hence, from Lemma 2, it follows that the equilibrium reached coincides with Eq. (7). □

Theorem 3. *If $A^* = \Psi\Delta\Phi$, $\|\mathbf{c}^*(t)\|$ is bounded and $\gamma_i < 1$ for $i = 1, \ldots, n$, then the system defined by Eq. (31) is stable.*

Proof. According to Lemma 3, if the free evolution is stable, then a bounded $\|\mathbf{c}^*(t)\|$ means that the overall system is stable. Using Gershgorin's Circle Theorem, it follows that the eigenvalues of the dynamic matrix lie (in the complex plane) within the union of circles centered at $-h_{ii}$ with radius equal to $|h_{ii}\rho_i|$ for $i = 1, \ldots, n(m+1)$, where:

$$h_{ii} = \begin{cases} k_{ii}, & \text{if } i = 1, \ldots, n\\ w_{ii}, & \text{if } i = n+1, \ldots, n(m+1)\end{cases} \tag{33}$$

and

$$\rho_i = \begin{cases} \bar{\gamma}_i, & \text{if } i = 1, \ldots, n\\ \sum_{j=1}^{nm} \phi_{i-n}^j, & \text{if } i = n+1, \ldots, n(m+1).\end{cases} \tag{34}$$

A sufficient condition for the stability of the system is that $\rho_i \leq 1$ for $i = 1, \ldots, n(m+1)$. Since, in the second case $\rho_i = \phi_i^{j*} \leq 1$, where ϕ_i^{j*} is the sole non-zero entry of the considered row of the dynamic matrix, the condition has to be verified only for $i = 1, \ldots, n$. By Theorem 2, it is sufficient that $\gamma_i \leq 1$ for $i = 1, \ldots, n$. □

The AB-IIM is thus a flexible, fine-grained, IIM-based framework. Since parameter tuning is mainly related to the production, transportation and consumption of resources, it is relatively straightforward to encode the knowledge of the various stakeholders. Another key property is that the derivation of the (fine-grained) Leontief coefficients, enables the AB-IIM to be transformed into a simple agent-based IIM. Moreover, the lack of economic data is more than compensated for by the availability of sector-specific knowledge and experience that can be utilized to obtain a realistic model.

5. Conclusions

The AB-IIM captures infrastructure interdependencies by decomposing each infrastructure into a set of interconnected elements and considering the exchange of resources between them. The model engages three matrices that represent the production, consumption and transmission/transportation of different resources. This characteristic is very useful because it facilitated the collection of data from asset owners, operators and domain experts in order to validate the model.

Another key feature is that the AB-IIM framework can be reduced to an agent-based IIM, where the Leontief coefficients are derived in a realistic manner. This is important because of the formality of the IIM and the fact that it is widely used to model and analyze interdependent critical infrastructures.

Our future research will attempt to incorporate a dynamic failure propagation component, where failures with different origins spread across a network. We will also continue our efforts focused at developing sophisticated frameworks that support the modeling and analysis of real-world scenarios with high fidelity.

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